

# The Netrinos of the Neighboring Brane

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## Abstract

The phenomenon of neutrino oscillations is studied usually as a mixing between the flavor neutrinos and the neutrinos having a definite mass. The mixing angles and the mass eigenvalues are treated independently in order to accommodate the experimental data. We suggest that neutrino oscillations are connected to the structure of spacetime. We expand on a recently proposed model, where two “mirror” branes coexist. One brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles. Majorana-type couplings mixes neutrinos in an individual brane, while Dirac-type couplings mixes neutrinos across the branes. We first focus our attention in a single brane. The mass matrix, determined by the Majorana mass, leads to mass eigenstates and further to mixing angles identical to the mixing angles proposed by the tri-bimaximal mixing. When we include the Dirac-type coupling, connecting the two branes, we obtain a definite prediction for the transition to a sterile neutrino (right-handed neutrino). With  $m_L$  ( $m_R$ ) the Majorana mass for the left (right) brane, we are able to explain the solar and the atmospheric neutrino data with  $m_L = 2m_R$  and  $m_R = 10^{-2}$  eV.

Neutrinos, more than 80 years from their inception, remain enigmatic. A number of experiments have helped us to determine their mixing angles and the scale of their masses [1]. Yet, we are lacking a satisfactory explanation for the nature of neutrinos, their number and the actual values of the parameters involved. In the present paper we attempt to connect the neutrino issues with the fundamental problem of theoretical physics, namely the structure of spacetime.

The entire universe (matter and radiation, stars, galaxies) is under a continuous evolution. Should we evolve also our own notions of space and time, should we look for a dynamical emergence of spacetime? Recently, by using the Cartan-Penrose connection of spinors to geometry, we explored the geometrical structures consistent with the quantum entanglement of two spinors [2]. Let us remind the thrust of the argument.

Relational logic, or its equivalent formulation as category theory, has been presented as the common foundation of quantum mechanics and string theory [3]. With relation (or a morphism) represented by a spinor [3, 4], we adopted the Cartan method of using spinors to obtain linear representations of geometries [5]. A single spinor gives rise to the Riemann-Bloch sphere, which is topologically equivalent to the null cone of Minkowski spacetime [6]. It is quite natural then to wonder what kind of geometry we obtain, when we entangle two spinors.

There are two ways to couple two spinors. The first recipe is coming from Majorana [7]. Given a left-handed spinor  $|\psi_L\rangle$ , we may construct a right-handed spinor  $|\chi_R\rangle$  by

$$|\chi_R\rangle = \sigma_2 |\psi_L\rangle^* \quad (1)$$

Starting with two independent left-handed Weyl spinors, we may induce a coupling between them by establishing a four-component Majorana spinor

$$|\Psi_M\rangle = \begin{pmatrix} |\chi_L\rangle \\ \sigma_2 |\psi_L\rangle^* \end{pmatrix} \quad (2)$$

Defining  $X_i = \langle \Psi_M | \gamma_i | \Psi_M \rangle$  ( $i = 0, 1, 2, 3$ ) we find that  $X_i$  is not a null vector [2]

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 = M_M^2 \quad (3)$$

with

$$\begin{aligned} X_4 &= i \langle \Psi_M | \Psi_M \rangle \\ X_5 &= \langle \Psi_M | \gamma_5 | \Psi_M \rangle \\ M_M^2 &= - (X_4^2 + X_5^2) \end{aligned} \quad (4)$$

Thus among two left-handed Weyl spinors (or two right-handed Weyl spinors), the Majorana's coupling induces a mass term.

The Dirac coupling involves a left-handed Weyl spinor and a right-handed Weyl spinor. Writing

$$|\Psi_D\rangle = \begin{pmatrix} |\chi_L\rangle \\ |\psi_R\rangle \end{pmatrix} \quad (5)$$

we obtain

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 = -M_D^2 \quad (6)$$

with

$$M_D^2 = (X_4^2 + X_5^2) \quad (7)$$

Let us define  $T = X_0$ ,  $t = M_D$ . The Dirac entanglement, equ.(6), takes the form of a space-like hyperboloid

$$T^2 - \sum_{i=1}^3 X_i^2 = t^2 \quad (8)$$

A comparison with the null cone geometry, indicates that quantum entanglement, specified and quantified by  $t$ , generates an extra dimension. The distance along this extra dimension indicates how far we are from the null cone. Furthermore our space-time acquires a double-sheet structure, reminding the ekpyrotic model where two branes co-exist [8, 9, 10]. There is though a distinct difference. In our model, by construction, one brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles.

The conventional way to restore left-right symmetry is to introduce an extra  $SU(2)_R$  gauge group in the energy desert above the scale of the standard  $SU(2)_L$  interactions. The right-handed gauge bosons are more massive compared to the left-handed gauge bosons, leading to parity violation at low energies [11, 12]. Within our approach the left-right symmetry is achieved with the extra dimension hosting two “mirror” branes, a left-handed brane and a right-handed brane. The most prominent candidate for mediation between the two branes is the neutrino particle. The left-handed neutrino, an essential ingredient of the standard model, resides in our brane, while its counterpart, the right-handed neutrino, resides in the other brane. Within our approach neutrino oscillations acquire a novel character. Majorana-type coupling mixes the left-handed flavor neutrinos residing in our brane, as well as the right-handed neutrinos residing in the other brane. Dirac-type coupling connects the left-handed neutrinos of our brane to the right-handed neutrinos of the other brane. From our point of view, right-handed neutrinos appear as sterile neutrinos, and the transition flavor neutrino - sterile neutrino - flavor neutrino amounts to a swapping between the two branes. Let us study first the mixing among the left-handed neutrinos, or focus our attention into our brane.

**i) single brane** We assume a “democratic principle” attributing the same value to all Majorana mass couplings. Then the mass matrix for the left-handed neutrinos will take the form

$$M = \begin{pmatrix} 0 & m & m \\ m & 0 & m \\ m & m & 0 \end{pmatrix} \quad (9)$$

The eigenvalues of  $M$ , involving a double root, are

$$\lambda_1 = \lambda_3 = -m \quad \lambda_2 = 2m \quad (10)$$

The corresponding eigenvectors are

$$\begin{aligned} N_1^T &= \frac{1}{\sqrt{6}} (2, -1, -1) \\ N_2^T &= \frac{1}{\sqrt{3}} (1, 1, 1) \\ N_3^T &= \frac{1}{\sqrt{2}} (0, 1, -1) \end{aligned} \quad (11)$$

Expressing the flavor left-handed neutrinos in terms of the mass eigenstates we write

$$|\nu_{f_i}\rangle = \sum_j c_{ij} |N_j\rangle \quad (12)$$

with  $\nu_{f_1}, \nu_{f_2}, \nu_{f_3}$  denoting respectively  $\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}$ .

Defining  $(U)_{ij} = c_{ij}$  we find that the mixing matrix  $U$  is

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad (13)$$

This type of mixing defines the celebrated tri-bimaximal mixing (TB mixing) [13-16]. We notice that the TB mixing has been proposed in order to accomodate the experimental data, while in our case emerges as the outcome of a Majorana-type coupling among the left-handed neutrinos.

Let us consider an initial flavor  $\nu_{e_L}$  beam. The transitions to other flavors are given by

$$P(\nu_e \rightarrow \nu_\mu) = P(\nu_e \rightarrow \nu_\tau) = \frac{4}{9} \sin^2 \left( \frac{3 m^2}{4 \hbar E} t \right) \quad (14)$$

Also

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{9} \left[ 9 - \frac{8}{9} \sin^2 \left( \frac{3 m^2}{4 \hbar E} t \right) \right] \quad (15)$$

The oscillations depend upon a single mass scale and clearly cannot reproduce the available data. The introduction of the right-handed brane allows us to have access to two more scales, the Majorana mass coupling in the right-handed brane and the Dirac mass coupling among the branes. We move then to the case of the two “mirror” branes.

**ii) mirror branes** On general grounds we expect the Majorana mass coupling in the right-handed brane to be of the same order of magnitude with the corresponding parameter in the left-handed brane. For general purposes we denote them by  $m_L, m_R$ , with the obvious correspondence. Each single left-handed neutrino, residing in our brane, is connected to all the right-handed neutrinos, residing in the other brane, by the same universal Dirac mass coupling  $\mu$ . Then the mass matrix involving the 6 neutrino states (3 left-handed plus 3 right-handed) will have the form

$$\mathcal{M} = \begin{pmatrix} M_L & M_+ \\ M_+ & M_R \end{pmatrix}. \quad (16)$$

$M_L (M_R)$  is a mass matrix identical to  $M$ , equ. (9), with  $m$  replaced by  $m_L (m_R)$ .  $M_+$  involves the mass terms connecting the two branes and is given by

$$M_+ = \begin{pmatrix} \mu & \mu & \mu \\ \mu & \mu & \mu \\ \mu & \mu & \mu \end{pmatrix} \quad (17)$$

The eigenvalues, involving two double roots, are

$$\begin{aligned} \lambda_1 &= \lambda_3 = -m_L \\ \lambda_4 &= \lambda_6 = -m_R \\ \lambda_2 &= (m_L + m_R) + [(m_L - m_R)^2 + 9\mu^2]^{\frac{1}{2}} \\ \lambda_5 &= (m_L + m_R) - [(m_L - m_R)^2 + 9\mu^2]^{\frac{1}{2}} \end{aligned} \quad (18)$$

Let us define

$$\begin{aligned} d &= [(m_L - m_R)^2 + 9\mu^2]^{\frac{1}{2}} \\ \delta_{\pm} &= d \pm (m_L - m_R) \end{aligned} \quad (19)$$

$$\cos \phi = \left( \frac{\delta_+}{2d} \right)^{\frac{1}{2}} \quad \sin \phi = \left( \frac{\delta_-}{2d} \right)^{\frac{1}{2}} \quad (20)$$

The corresponding eigenvectors are

$$\begin{aligned} N_1^T &= \frac{1}{\sqrt{6}} ( 2, -1, -1, 0, 0, 0 ) \\ N_2^T &= \frac{1}{\sqrt{3}} ( \cos \phi, \cos \phi, \cos \phi, \sin \phi, \sin \phi, \sin \phi ) \\ N_3^T &= \frac{1}{\sqrt{2}} ( 0, 1, -1, 0, 0, 0 ) \\ N_4^T &= \frac{1}{\sqrt{6}} ( 0, 0, 0, 1, -2, 1 ) \\ N_5^T &= \frac{1}{\sqrt{3}} ( \sin \phi, \sin \phi, \sin \phi, -\cos \phi, -\cos \phi, -\cos \phi ) \\ N_6^T &= \frac{1}{\sqrt{2}} ( 0, 0, 0, -1, 0, 1 ) \end{aligned} \quad (21)$$

The similarities and the differences with the case of a single brane, equ. (11), are apparent.  $N_1, N_3 (N_4, N_6)$  involve mixing within the individual left (right) brane.  $N_2$  and  $N_5$  connect the two branes. For  $m_L = m_R$ ,  $\phi = \frac{\pi}{4}$  and the two branes are equally present in the mixing phenomenon. For small Dirac coupling compared to the Majorana couplings we obtain  $\phi \simeq 0$ .

The mixing matrix connecting the flavor eigenstates (left-handed and right-handed)

to the six eigenvectors takes the form

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \cos \phi & 0 & 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \cos \phi & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \cos \phi & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 \\ 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \cos \phi & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 & -\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} \cos \phi & 0 \\ 0 & \frac{1}{\sqrt{3}} \sin \phi & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \cos \phi & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (22)$$

Again for  $\phi = 0$  the upper left part of the matrix gives the previous result, equ. (13), for the single brane.

Imagine that at  $t = 0$  we start with a pure  $\nu_{eL}$  beam. The probability to find later another flavor is given by

$$P(\nu_{eL} \rightarrow \nu_{\mu L}) = P(\nu_{eL} \rightarrow \nu_{\tau L}) = \frac{1}{9} \left\{ 1 + \cos^4 \phi + \sin^4 \phi + 2 \left[ \cos^2 \phi \sin^2 \phi \cos \frac{(\omega_+ - \omega_-) t}{2\hbar E} - \cos^2 \phi \cos \frac{\omega_+ t}{2\hbar E} - \sin^2 \phi \cos \frac{\omega_- t}{2\hbar E} \right] \right\} \quad (23)$$

where

$$\begin{aligned} \omega_+ &= m_L^2 + 2m_R^2 + 9\mu^2 + 2d(m_R + m_L) \\ \omega_- &= m_L^2 + 2m_R^2 + 9\mu^2 - 2d(m_R + m_L) \\ \omega_+ - \omega_- &= 4d(m_R + m_L) \end{aligned} \quad (24)$$

The transition to a generic sterile neutrino (an incoherent sum of all right-handed neutrinos) is given by

$$P(\nu_e \rightarrow \nu_s) = \frac{1}{9} \sin^2 2\phi \sin^2 \left( \frac{1}{4\hbar E} (\omega_+ - \omega_-) t \right) \quad (25)$$

Notice that for the transition of the  $\nu_{\mu L}$  we find

$$P(\nu_{\mu L} \rightarrow \nu_{eL}) = P(\nu_{\mu L} \rightarrow \nu_{\tau L}) = P(\nu_{eL} \rightarrow \nu_{\mu L}) \quad (26)$$

We may recall the neutrino oscillation data [17]. Solar and atmospheric neutrino oscillations define two distinct mass scales

$$\Delta m_s^2 \simeq 5 * 10^{-5} eV^2 \quad \Delta m_a^2 \simeq 2 * 10^{-3} eV^2 \quad (27)$$

A neutrino oscillation experiment defines a specific value for the parameter  $\frac{t}{E}$  (the distance traveled by the neutrino over its energy). Large values of  $\frac{t}{E}$  allow to explore small values of  $\Delta m^2$ , or correspondingly small  $\omega$ . Solar neutrinos correspond to low energy neutrinos covering huge distance, therefore their oscillation is determined by  $\omega_-$ . Atmospheric neutrinos involve higher energies and smaller distances and their oscillation is controlled by  $\omega_+$ . Accordingly we assign

$$\begin{aligned} \omega_- &\simeq \Delta m_s^2 \\ \omega_+ &\simeq \Delta m_a^2 \end{aligned} \quad (28)$$

There is a conflicting evidence for the existence of a sterile neutrino [18]. At any rate the amplitude for a transition to a sterile neutrino is expected to be small and correspondingly  $\sin \phi$ , see equ. (25), and the Dirac coupling  $\mu$  are small. Adopting the hierarchy  $(m_L - m_R) > \mu$  we find that the values

$$m_L \simeq 2m_R \quad m_R \simeq 10^{-2} eV \quad (29)$$

reproduce the observed scales, equ. (28). The precise smallness of  $\mu$  will fix the magnitude of  $\sin \phi$  and therefore the probability to a sterile neutrino oscillation. Notice however that within our scheme the mass scale for the transition to a sterile neutrino is at a sub-eV scale ( $3 * 10^{-2}$  eV), rather far from the value suggested by the LSND experiment.

The conventional approach to the phenomenon of neutrino oscillations is to consider it as a manifestation of a mixing between the flavor eigenstates and the mass eigenstates. The mixing angles and the masses of the mass eigenstates are treated independently and are determined largely by the experimental data. There is also an effort to accommodate the available data by making appeal to discrete groups [19]. We offer an alternative approach, by proposing that neutrino oscillations are connected to the structure of space-time. Space-time hosts two branes, one brane where the left-handed particles reside (our brane) and another brane where the right-handed particles reside. The long sought left-right symmetry is achieved through the geometry of space-time. Majorana-type couplings connect the neutrinos living in an individual brane, while Dirac-type couplings connect neutrinos across the branes. We managed to treat at the same time both the masses involved and the mixing angles, by making appeal to first principles. Is this success fortuitous? We may argue that it is a sign for the existence of “mirror” branes. But clearly further indications are needed.

Finally we would like to remind the experimental evidence for a small non-vanishing value for the matrix element  $c_{13}$  [20-23]. It seems that this small value indicates a hidden substructure and work along this line is in progress.

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